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NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION

CLASS - 12 (PCM)
Question Paper Code : UN484

KEY

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SOLUTIONS

MATHEMATICS

01. (C) $f(x) = \dfrac{|x|-1}{|x|+1}$

for one-one function if $f(x_1) = f(x_2)$ then x_1 must be equal to x_2

Let $f(x_1) = f(x_2)$

$$\frac{|x_1|-1}{|x_1|+1} = \frac{|x_2|-1}{|x_2|+1}$$

$$|x_1||x_2| + |x_1| - |x_2| - 1$$

$$= |x_1||x_2| - |x_1| + |x_2| - 1$$

$$|x_1| - |x_2| = |x_2| - |x_1|$$

$$2|x_1| = 2|x_2|$$

$$|x_1| = |x_2|$$

$$x_1 = x_2,\ x_1 = -x_2$$

02. (C) $2\pi - \left(\sin^{-1}\dfrac{4}{5} + \sin^{-1}\dfrac{5}{13} + \sin^{-1}\dfrac{16}{65}\right)$

$$= 2\pi - \left(\tan^{-1}\frac{4}{3} + \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{16}{63}\right)$$

$$\left(\because\ \sin^{-1}\frac{4}{5}=\tan^{-1}\frac{4}{3}\right)$$

$$= 2\pi - \left\{\tan^{-1}\left(\frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}\frac{5}{12}}\right)+\tan^{-1}+\frac{16}{63}\right\}$$

$$= 2\pi - \left(\tan^{-1}\frac{63}{16}+\tan^{-1}+\frac{16}{63}\right)$$

$$= 2\pi - \left(\tan^{-1}\frac{63}{16}+\cot^{-1}+\frac{63}{16}\right)$$

$$= 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$$

03. (D) Solution of $x^2 + x + 1 = 0$ is ω, ω^2

So, $\alpha = \omega$ and

$\omega^4 = \omega^3.\omega = 1.\omega = \omega$

$$A^2 = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & \omega & \omega^2\\ 1 & \omega^2 & \omega\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}$$

$A^4 = I$

$A^{31} = A^{28} \times A^3 = A^3$

04. (C) $$\Delta = \begin{vmatrix}x-2 & 2x-3 & 3x-4\\ 2x-3 & 3x-4 & 4x-5\\ 3x-5 & 5x-8 & 10x-17\end{vmatrix}$$

$$\begin{bmatrix}C_3 \to C_3 - C_2\\ C_2 \to C_2 - C_1\end{bmatrix}$$

$$\Delta = \begin{vmatrix}x-2 & x-1 & x-1\\ 2x-3 & x-1 & x-1\\ 3x-5 & 2x-3 & 5x-9\end{vmatrix}$$

$$[R_2 \to R_2 - R_1]$$

$$\Delta = \begin{vmatrix}x-2 & x-1 & x-1\\ x-1 & 0 & 0\\ 3x-5 & 2x-3 & 5x-9\end{vmatrix}$$

$\Delta = -(x-1)[(x-1)(5x-9)-(x-1)(2x-3)]$

$\Delta = -(x-1)[(x-1)(5x^2-14x+9)-(2x^2-5x+3)]$

$= -3x^3 + 12x^2 - 15x + 6$

So, B + C = –3

05. (A) $C_1 \to C_1 + C_2$

$$\begin{vmatrix}2 & \sin^2\theta & 4\cos6\theta\\ 2 & 1+\sin^2\theta & 4\cos6\theta\\ 1 & \sin^2\theta & 1+4\cos6\theta\end{vmatrix}=0$$

$R1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

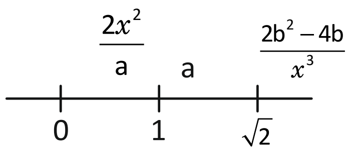
$$\begin{vmatrix}0 & -1 & 0\\ 1 & 1 & -1\\ 1 & \sin^2\theta & (1+4\cos6\theta)\end{vmatrix}=0$$

On expanding, we get $2 + 4\cos6\theta = 0$

$$\cos6\theta=-\frac{1}{2}\ \because\theta\in\left(0,\frac{\pi}{3}\right)\Rightarrow6\theta(0,2\pi)$$

Therefore, $6\theta=\frac{2\pi}{3}\text{or}\frac{4\pi}{3}\Rightarrow\theta=\frac{\pi}{9}\text{or}\frac{2\pi}{9}$

06. (C) Continuity at $x = 1$



$$\frac{2}{a}=a\Rightarrow a=\pm\sqrt{2}$$

Continuity at $x = \sqrt{2}$ $a=\sqrt{2}$

$$a=\frac{2b^2-4b}{2\sqrt{2}}$$

Put a = $\sqrt{2}$

$2 = b^2 - 2b \Rightarrow b^2 - 2b - 2 = 0$

$$b=\frac{2\pm\sqrt{4+4.2}}{2}=1\pm\sqrt{3}$$

$$\text{So, }(a,b)=\left(\sqrt{2},1-\sqrt{3}\right)$$

07. (B) $f(x) = 2x^3 + ax^2 + bx$

let, a = –1, b = 1

Given that $f(x)$ satisfy Rolle's theorem in interval [–1, 1]

$f(x)$ must satisfy two canditions.

(1), f(a) = f(b)

(2), f'(c) = 0 (c should be between a and b)

$f(a) = f(1) = 2(1)^2 + a(1)^2 + b(1)$

$= 2 + a + b$

$f(b) = f(-1) = 2(-1)^2 + a(-1)^2 + b(-1)$

$= -2 + a - b$

$f(a) = f(b)$

$2 + a + b = -2 + a - b$

$2b = -4$

$b = -2$

(given that $c = \frac{1}{2}$)

$f'(x) = 6x^2 + 2ax + b$

at $x = \frac{1}{2}$, $f'(x) = 0$

$$0 = 6\left(\frac{1}{2}\right)^2 + 2a\left(\frac{1}{2}\right) + b$$

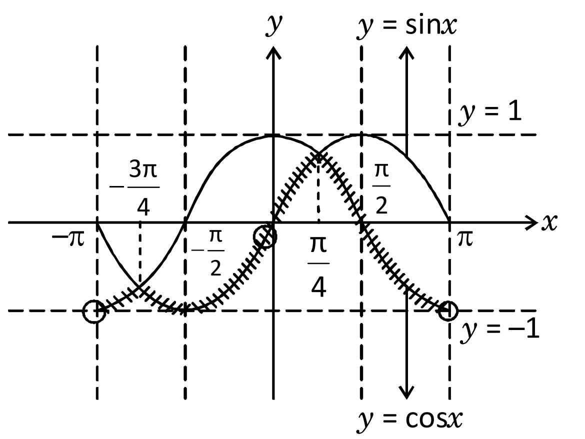
$$\frac{3}{2} + a + b = 0$$

$$\frac{3}{2} + a - 2 = 0$$

$$a = 2 - \frac{3}{2} = \frac{1}{2}$$

$$2a + b = 2 \times \frac{1}{2} - 2 = 1 - 2 = -1$$

08. (B) $f(x) = \min(\sin x, \cos x)$



$\because$ $f(x)$ is not differentiable at $x = \frac{-3\pi}{4}, \frac{\pi}{4}$

$$\therefore S = \left\{-\frac{3\pi}{4}, \frac{\pi}{4}\right\}$$

$$S \subseteq \left\{-\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4}\right\}$$

09. (D) Since, function $f(x)$ is continuous at x = 1, 3

$f(1) = f(1^+)$

$ae + be^{-1} = c$ ______(1)

$f(3) = f(3^+)$

$9c = 9a + 6c$

$c = 3a$ ______(2)

From (i) and (ii),

$b = ae(3 - e)$ ______(3)

$$\begin{cases} ae^{x} + be^{-x} & -1 < x < 1 \\ 2cx & 1 < x < 3 \\ 2ax + 2c & 3 < x < 4 \end{cases}$$

$f'(0) = a - b$, $f'(2) = 4c$

Given, $f'(0) + f'(2) = e$

$a - b + 4c = e$ ______(4)

From equs, (1), (2), (3) and (4),

$a - 3ae + ae^2 + 12a = e$

$13a - 3ae + ae^2 = e$

$$a = \frac{e}{e^2 - 3e + 13}$$

10. (A) $\int\left(\frac{x}{x\sin x+\cos x}\right)^2$

$\frac{d}{dx}(x\sin x+\cos x) = x\cos x$

$= \int \frac{x\cos x}{(x\sin x+\cos x)^2}\left(\frac{x}{\cos x}\right)dx$

$= \frac{x}{\cos x}\left[\frac{-1}{x\sin x+\cos x}\right]$

$-\frac{x\sin x+\cos x}{\cos^2 x}\left[\frac{-1}{x\sin x+\cos x}\right]dx$

$= \frac{x}{\cos x}\left[\frac{-1}{x\sin x+\cos x}\right] + \int \sec^2 x\,dx$

$= \frac{-x\sec x}{x\sin x+\cos x} + \tan x + C$

11. (D) $l = \int_1^e \left\{\left(\frac{x}{e}\right)^{2x} - \left(\frac{e}{x}\right)^x\right\}\log_e x\,dx$

Let $\left(\frac{x}{e}\right)^x = t$

$x\,ln\left(\frac{x}{e}\right) = ln\,t$

$x\,(ln\,x - 1) = ln\,t$

On differentiating both sides w.r.t. x we get

$lnx.\,dx = \frac{dt}{t}$

When x = e then t = 1 and when x = 1

then $t = \frac{1}{e}$

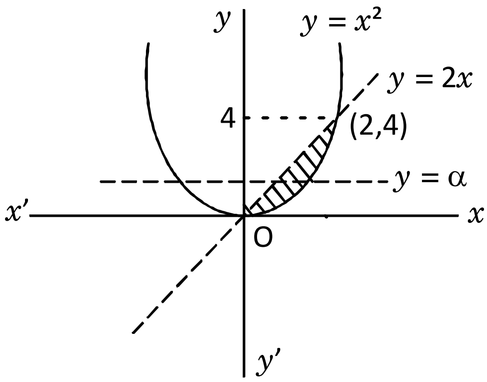
$I = \int_{\frac{1}{e}}^{1}\left(t^2-\frac{1}{t}\right)\frac{dt}{t} = \int_{\frac{1}{e}}^{1}\left(t-\frac{1}{t^2}\right)dt$

$= \left(\frac{t^2}{2}+\frac{1}{t}\right)_{\frac{1}{e}}^{1} = \left(\frac{1}{2}+1\right) - \left(\frac{1}{2e^2}+e\right)$

$= \frac{3}{2} - e - \frac{1}{2e^2}$

12. (B) Let $y = x^2$ and $y = 2x$

According to question



$\therefore \int_0^{\alpha}\left(\sqrt{y}-\frac{y}{2}\right)dy = \int_{\alpha}^{4}\left(\sqrt{y}-\frac{y}{2}\right)dy$

$\left[\frac{y^{3/2}}{\frac{3}{2}}\right]_0^{\alpha} - \left[\frac{y^2}{4}\right]_0^{\alpha} = \left[\frac{y^{3/2}}{\frac{3}{2}}\right]_{\alpha}^{4} - \left[\frac{y^2}{4}\right]_{\alpha}^{4}$

$\frac{2}{3}\alpha^{3/2} - \frac{\alpha^2}{4} = \frac{2}{3}\left(8-\alpha^{3/2}\right) - \frac{1}{4}\left(16-\alpha^2\right)$

$\frac{4}{3}\alpha^{3/2} - \frac{\alpha^2}{2} = \frac{4}{3}$

$8\alpha^{3/2} - 3\alpha^2 = 8$

$\therefore 3\alpha^2 - 8\alpha^{3/2} + 8 = 0$

13. (C) The given differential eqn, is

$\frac{dy}{\sqrt{1-y^2}} + \frac{dx}{\sqrt{1-x^2}} = 0$

$\sin^{-1}y + \sin^{-1}x = c$

At $x = \frac{1}{2}, y = \frac{\sqrt{3}}{2}$

$c = \frac{\pi}{2}$

$\sin^{-1}y = \cos^{-1}x$

Hence, $y\left(-\frac{1}{\sqrt{2}}\right) = \sin\left(\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)\right)$

$= \sin\left(\pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right) = \frac{1}{\sqrt{2}}$

14. (D) $\because \bar{a}, \bar{b}$ and $\bar{c}$ are coplanar

$$\therefore \begin{vmatrix} 1 & 2 & 4 \\ 1 & \lambda & 4 \\ 2 & 4 & (\lambda^2-1) \end{vmatrix} = 0$$

$\lambda^3 - \lambda - 16 + 2(8 - \lambda^2 + 1) + 4(4 - 2\lambda) = 0$

$\lambda^3 - 2\lambda^2 - 9\lambda + 18 = 0$

$(\lambda - 2)(\lambda - 3)(\lambda + 3) = 0$

$\lambda = 2, \quad \bar{c} = 2\hat{i} + 4\hat{j} + 3\hat{k}$

$$\therefore \bar{a} \times \bar{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 4 \\ 2 & 4 & 3 \end{vmatrix} = -10\hat{i} + 5\hat{j}$$

For $\lambda = 3$ or -3, $\bar{c} = 2\bar{a}$

$\bar{a} \times \bar{c} = 0$ (Rejected)

15. (A) $L_1 = \bar{r} = (\hat{i} - \hat{j}) + l(2\hat{i} + \hat{k})$

$L_2 = \bar{r} = (2\hat{i} - \hat{j}) + m(\hat{i} + \hat{j} - \hat{k})$

Equating coeff. of $\hat{i}$, $\hat{j}$ and $\hat{k}$ of L_1 and L_2

$2l + 1 = m + 2$ _______(1)

$-1 = -1 + m$

$m = 0$ _______(2)

$l = -m$ _______(3)

$m = l = 0$, which is not satisfy eqn, (i) hence lines do not intersect for any value of l and m

16. (D) Probability of sum getting 6, P(A) = $\frac{5}{36}$

Probability of sum getting 7, P(B) = $\frac{6}{36}$

$= \frac{1}{6}$

P(A wins) = P(A) + P($\bar{A}$)P($\bar{B}$)P(A)

+ P($\bar{A}$).P($\bar{B}$)P($\bar{A}$)P($\bar{B}$)P(A) + ...

$\frac{5}{36} + \left(\frac{31}{36}\right)\left(\frac{30}{36}\right)\left(\frac{5}{36}\right) + \ldots\ldots \infty$

$\frac{5}{36}\left(1 + \frac{155}{216} + \left(\frac{155}{216}\right)^2 + \ldots\ldots\infty\right)$

$\dfrac{\frac{5}{36}}{\frac{61}{216}} = \frac{30}{61}$ $\left(\because s_\infty = \frac{a}{1-r}\right)$

17. (D) $\sin x$ is a periodic function with period 2π

$\sin x$ is a many one function.

18. (A) If $\mathrm{Tan}^{-1} 2 = \alpha$ then $\mathrm{Tan}\ \alpha = 2$.

If $\mathrm{Tan}^{-1}\frac{4}{3} = \beta$ then $\tan\beta = \frac{4}{3}$

$\cos\beta = \frac{3}{5}$

$x = \sin(2\mathrm{Tan}^{-1} 2) = \sin 2\alpha = \frac{2\tan\alpha}{1+\tan^2\alpha}$

$= \frac{4}{1+4} = \frac{4}{5}$

$y = \sin\left(\frac{1}{2}\mathrm{Tan}^{-1}\frac{4}{3}\right) = \sin\left(\frac{\beta}{2}\right)$

$= \frac{\sqrt{1-\cos\beta}}{2} = \frac{\sqrt{1-\frac{3}{5}}}{2} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

$\therefore x > y$

19. (B) Let A_1, A_2, A_3 denote the events that the bag contains 4, 5, 6 red balls respectively and E be the event that four red balls are drawn from the bag. Now $P(A_1) = P(A_2) = P(A_3) = \frac{1}{3}$

$P(E|A_1) = \frac{{}^4C_4}{{}^6C_4} = \frac{1}{15}$, $P(E|A_2) = \frac{{}^5C_4}{{}^6C_4}$

$= \frac{5}{15}$ $P(E|A_3) = \frac{{}^6C_4}{{}^6C_4} = \frac{15}{15}$

$P(A_2|E) = \frac{P(A_2)P(E|A_2)}{\Sigma P(A_1)P(E|A_1)}$

$\dfrac{\frac{1}{3}\times\frac{5}{15}}{\frac{1}{3}\times\frac{1}{15}+\frac{1}{3}\times\frac{5}{15}+\frac{1}{3}\times\frac{15}{15}} = \frac{5}{1+5+15}$

$= \frac{5}{21}$

20. (A) $\dfrac{a.(b\times c)}{(c\times a).b}+\dfrac{b.(a\times c)}{c.(a\times b)}$

$= \dfrac{[a\ \ b\ \ c]}{[c\ \ a\ \ b]}+\dfrac{[b\ \ a\ \ c]}{[c\ \ a\ \ b]}$

$= \dfrac{[a\ \ b\ \ c]}{[a\ \ b\ \ c]}-\dfrac{[a\ \ b\ \ c]}{[a\ \ b\ \ c]}$

$= 1 - 1 = 0$

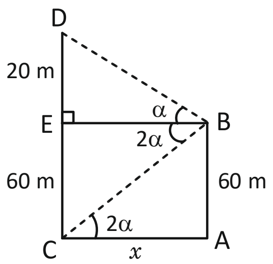
21. (A) Let AB be the tower T_1 with foot at A.

Let CD be the tower T_2 with foot at C

Let E be the projection of B on CD.

Let $AC = BE = x$, $\angle EBD = \theta$

Then, $\angle CBE = \angle ACB = 2\theta$



Given AB = 60 m, CD = 80 m.

Then ED = 20 m

From ΔBDE, $\tan\theta = \dfrac{20}{x}$

From ΔABC, $\tan 2\theta = \dfrac{60}{x}$

$\dfrac{2\tan\theta}{1-\tan^2\theta} = \dfrac{60}{x}$

$\dfrac{2\left(\dfrac{20}{x}\right)}{1-\dfrac{400}{x^2}} = \dfrac{60}{x}$

$2 = 3\left(1-\dfrac{400}{x^2}\right)$

$2x^2 = 3x^2 - 1200$

$x^2 = 1200$

$x = 20\sqrt{3}$

22. (B) $y\,dx - (x + 2y^2)\,dy = 0$

$\dfrac{y\,dx - x\,dy}{y^2} = 2dy$

$d\left(\dfrac{x}{y}\right) = 2dy$

$\int d\left(\dfrac{x}{y}\right) = 2\int dy$

$\dfrac{x}{y} = 2y + c$

$x = 2y^2 + cy$

$f(y) = 2y^2 + cy$ $\quad[\because \text{Given } x = f(y)]$

Given $f(-1) = 1$

$1 = 2 - c$

$c = 1$

$\therefore f(y) = 2y^2 + y$

$f(1) = 2 + 1 = 3$

23. (C) $I = \int_0^{\pi/2} \dfrac{\sin^3 x}{\sin x + \cos x}dx$

$= \int_0^{\pi/2} \dfrac{2\sin(\pi/2 - x) + 3\cos(\pi/2 - x)}{\sin(\pi/2 - x) + \cos(\pi/2 - x)}dx$

$= \int_0^{\pi/2} \dfrac{2\cos x + 3\sin x}{\sin x + \cos x}dx$

$\therefore 2I = \int_0^{\pi/2} \dfrac{5\sin x + 5\cos x}{\sin x + \cos x}dx$

$= 5\int_0^{\pi/2} \dfrac{\sin x + \cos x}{\sin x + \cos x}dx$

$= 5[x]_0^{\pi/2} = \dfrac{5\pi}{2} \quad \therefore I = \dfrac{5\pi}{4}$

24. (B) $\sin^2\left[\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right]$

$$= \frac{1}{\operatorname{cosec}^2\left[\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right]}$$

$$= \frac{1}{1+\cot^2\left[\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right]}$$

$$= \frac{1}{1+\frac{1-x}{1+x}}$$

$$= \frac{1+x}{1+x+1-x} = \frac{1+x}{2}$$

$$\therefore \frac{d}{dx}\left[\sin^2\left(\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right)\right] = \frac{1}{2}$$

25. (D) $\sim(\sim s \vee (\sim r \wedge s))$

$s \wedge (\sim(\sim r \wedge s)$

$s \wedge (r \vee \sim s)$

$(s \wedge r) \vee (s \wedge \sim s)$

$(s \wedge r) \vee F$ $(\because s \wedge \sim s$ is false)

$s \wedge r$

PHYSICS

26. (B) Molar mass of copper M = 63.5 gram = 63.5×10^{-3} kg

Density of copper $\rho = 9 \times 10^3$ Kg/m^3

No. of copper atoms per unit volume:

N = No. of moles in unit volume × No. of atoms in 1 mole (N_A) ...(1)

No. of moles in unit volume = $\frac{\text{Mass of unit volume}}{\text{Mass of one mole}}$

$$= \frac{\text{Density}}{\text{Molar Mass}} = \frac{\rho}{M}$$

Therefore, Using Equation (1), We get:

$N = \frac{\rho}{M} \times N_A$ Where $N_A = 6.023 \times 10^{23}$

$\Rightarrow N = \frac{9\times10^3 \times 6.023\times10^{23}}{63.5\times10^{-3}} = 8.54 \times 10^{28}$ m^{-3}

One copper atom contributes one conduction electron.

So, No. of conduction electrons per unit volume = No. of copper atoms per unit volume

$\therefore$ n = N = 8.54×10^{28} m^{-3}

Given: Current I = 1.5 A, Cross sectional area A = 1×10^{-7} m^2

We know that,

$I = neAv_d$

So, Drift Velocity $v_d = \frac{I}{neA}$

$\Rightarrow$ Vd =

$$\frac{1.5\text{ A}}{8.5\times10^{28} \times (1.6\times10^{-19}\text{ C}) \times (10^{-7}\text{m}^{-2})}$$

$= 1.1 \times 10^{-3}$ m/s

27. (C) As the current is in the phase with the applied voltage, X must be R.

$$R = \frac{V_0}{I_0} = \frac{200\ V}{5\ A} = 40\ \Omega$$

As current lags behind the voltage by 90°, Y must be an inductor.

$$X_L = \frac{V_0}{I_v} = \frac{200\ V}{5\ A} = 40\ \Omega$$

In the series combination of X and Y,

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{40^2 + 40^2} = 40\sqrt{2}\ \text{ohm}$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{V_0}{\sqrt{2}\,Z} = \frac{200}{\sqrt{2} \times (40\sqrt{2})} = \frac{5}{2} A$$

28. (B) Because of large permeability of soft iron, magnetic lines of force prefer to pass through it. Concentration of lines in soft iron bar increases as shown.

29. (D) Radius of the ring = a = 0.10 m

Total charge = q = 50 × 10^{-6} C

Distance = x = 0.10 m

(i) $$E = \frac{1}{4\pi E_0} \cdot \frac{qx}{(a^2 + x^2)^{3/2}}$$

$$= \frac{9 \times 10^9 \times 50 \times 10^{-6} \times 0.1}{(0.1^2 + 0.1^2)^{3/2}} = 1.59 \times 10^7\ \text{N/C}$$

(ii) When x = 100 cm = 1 m,

$$E = \frac{9 \times 10^9 \times 50 \times 10^{-6} \times 0.1}{(0.1^2 + 1^2)^{3/2}} = 4.45 \times 10^4\ \text{N/C}$$

30. (C) $$v = \frac{c}{\lambda} = c.R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$= 3 \times 10^8 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{9}{16} \times 10^{15}\ \text{Hz}.$$

31. (D) $$dB = \frac{\mu_0}{4\pi} \frac{Idl\sin\theta}{r^2}$$

$$= \frac{10^{-7} \times (5 \times 10^{-2}) \times \sin 45°}{(2)^2}$$

= 8.8 × 10^{-10} T vertically downwards

32. (A) The object and its image always move in opposite directions.

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Differentiating with respect to time, we get

$$-\frac{1}{v^2}\frac{dv}{dt} - \frac{1}{u^2}\frac{du}{dt} = 0$$

Let $\frac{dv}{dt} = V$ (Velocity of image)

$\frac{du}{dt} = U$ (Velocity of object)

then $\frac{V}{U} = -\frac{v^2}{u^2}$

The negative sign shows that V and U are always oriented in opposite directions irrespective of the nature of the mirror.

33. (A) Each atom of $_6C^{14}$ contains 6 p, 6 e and 8 n

∴ In 14 gram of $_6C^{14}$

p = 6 × 6 × 10^{23} = 36 × 10^{23}

p = 8 × 6 × 10^{23} = 48 × 10^{23}

e = p = 36 × 10^{23}

34. (B) The cut-off wavelength of the continuous X-rays does not depend on the atomic number of the target but it depends on accelerating potential applied to the anode.

35. (B) Radius of the small drop = r = 0.02 m

Radius of the bigger drop = R = ?

Volume of one bigger drop = Volume of 64 small drops

$$\frac{4}{3}\pi R^3 = 64 \times \frac{4}{3}\pi r^3$$

$R^3 = 4^3 r^3$

R = 4r = 4 × 0.02 = 0.08 m

Total charge on the bigger drop = q = 64 × 0.5 × 10^{-6} = 32 × 10^{-6} C

Potential at the surface of the bigger drop

$$= \frac{q}{4\pi\varepsilon_0 R} = 9 \times 10^9 \times \frac{32 \times 10^{-6}}{0.08}$$

= 3.6 × 10^6 V

36. (C) The electromagnetic wave being packets of energy moving with the speed of light may pass through the region.

37. (B) As, $\phi_0 = \frac{hc}{\lambda_0}$; so $\frac{\phi_{0_T}}{\phi_{0_{Na}}} = \frac{\lambda_{Na}}{\lambda_T}$

or $\lambda_T = \lambda_{Na} \times \frac{\phi_{0_{Na}}}{\phi_{0_T}} = \frac{5460 \times 2.3}{4.5} = 2791\ \text{Å}$

38. (D) $B_1 = \frac{\mu_0}{4\pi}\frac{2\pi ni}{r}$ and

$$B_2 = \frac{\mu_0}{4\pi}\frac{2\pi nir^2}{(r^2+h^2)^{3/2}} 80$$

$$\frac{B_2}{B_1} = \left(1+\frac{h^2}{r^2}\right)^{-3/2}$$

Fractional decrease in the magnetic field will be

$$= \frac{B_1 - B_2}{B_1} = \left(1 - \frac{B_2}{B_1}\right)$$

$$= \left[1 - \left(1+\frac{h^2}{r^2}\right)^{-3/2}\right]$$

$$= 1 - \left(1 - \frac{3h^2}{2r^2}\right) = \mathbf{\frac{3h^2}{2r^2}}$$

39. (B) From $s = ut + \frac{1}{2}at^2 = \frac{1}{2}at^2$ ($\because$ u = 0)

$t = \sqrt{\frac{2s}{a}}$ As s is same, $\because t \propto \frac{1}{\sqrt{a}}$

$$\frac{t_2}{t_1} = \sqrt{\frac{a_1}{a_2}} = \sqrt{\frac{q_e/M_e}{q_p/M_p}} = \sqrt{\frac{M_p}{M_e}}$$

40. (B) $e = \frac{Mdi}{dt} = \left(\frac{\mu_0 N_1 N_2 A}{l}\right)\frac{di}{dt}$

$$= \frac{4\pi \times 10^{-7} \times 2000 \times 300 \times 1.2 \times 10^{-3}(4)}{0.3 \times 0.25}$$

$= 4.8 \times 10^{-2}$ V

CHEMISTRY

41. (C) Concentration of $ZnSO_4$ solution = 0.1 M

Percentage of dissociation of $ZnSO_4$ solution = 95%

$\therefore$ Concentration of Zn^{2+} ions in $ZnSO_4$ solution =

$$\frac{0.1 \times 95}{100} = 0.095\ \text{M}$$

Thus, the electrode can be represented as :

Zn (s) | Zn^{2+} (aq. 0.095 M)

Reduction reaction taking place at this electrode is :

Zn^{2+} (aq. 0.095 M) + $2e^-$ $\longrightarrow$ Zn (s) (Here n = 2)

According to Nernst equation, the reduction potential of the above electrode $[(E_{red})_{elec}]$ is given by :

$$(E_{red})_{elec} = (E^o_{red})_{elect} - \frac{RT}{nF}\ln\frac{[\text{Products}]}{[\text{Reactants}]}$$

or

$$(E_{red})_{elec} = (E^o_{red})_{elect} - \frac{2.303\,RT}{nF}\log\frac{[\text{Products}]}{[\text{Reactants}]}$$

or

$$E_{Zn^{2+}/Zn} = E^o_{Zn^{2+}/Zn} - \frac{2.303\,RT}{nF}\log\frac{[Zn]}{[Zn^{2+}(aq)]}$$

or $$E_{Zn^{2+}/Zn} = -0.76 - \frac{2.303\,RT}{nF}\log\frac{1}{0.095}$$

$$= \frac{-0.76 + 2.303 \times 8.31 \times 298 \log 0.095}{2 \times 96500}$$

or $E_{Zn^{2+}/Zn}$ = – 0.79 Volt

42. (B) Fe^{3+}, Zn^{2+} and Cu^{2+} ions are present in slightly acidic solution. On adding 6 H NH_3 solution i.e., 6 M NH_4OH we get the following reactions :

Fe^{3+} + 3 OH^- $\longrightarrow$ $Fe(OH)_3$
Dark brown ppt.

$Zn^{2+} + 4\,NH_3 \longrightarrow [Zn(NH_3)_4]^{2+}$
Colourless solution

$Cu^{2+} + 4\,NH_3 \longrightarrow [Cu(NH_3)_4]^{2+}$
Deep blue solution

In this way dark brown ppt. of $Fe(OH)_3$ can be separated from Cu^{2+} and Zn^{2+} ammine complex solution in a single step by adding 6 M NH_3.

43. (C) A Compound given in option (C) is a 3° alcohol, it undergoes dehydration very easily.

44. (B) The rate law equation can be written as,

Rate = $k[CH_3CHO]^n$ Where n = order of reaction

Substituting the given data, we get,

$0.70 = k[300]^n$ ..... (i)

$0.31 = k[200]^n$ ..... (ii)

Dividing equation (i) by (ii)

$$\frac{0.70}{0.31} = \left(\frac{300}{200}\right)^n$$

Taking log on both sides

$$\text{or } \log\left(\frac{0.70}{0.31}\right) = n\log\left(\frac{300}{200}\right)$$

log 2.258 = n log 1.5

$$\text{or } n = \frac{\log 2.258}{\log 1.5} = \frac{0.3537}{0.1761}$$

∴ Order of reaction = 2.00

45. (A) In acidic solution, NH_3 forms a bond with H^+ to form NH_4^+ ion which does not have a lone pair on N to act as a ligand.

46. (B) Volume of water = 500 cm^3

Density of water = 0.997 g cm^{-3}

Mass of water = Volume × Density

= 500 × 0.997 = 498.5 g

Amount of acetic acid

$$= \frac{3.0 \times 10^{-3} \times 1000\,g}{60\,g\,mol^{-1}}$$

= 0.05 mol

Molality =

$$\frac{\text{Moles of acetic acid}}{\text{Mass of water in grams}} = 1000\,g/kg$$

$$= \frac{0.05 \times 1000}{498.5}\,mol\,kg^{-1} = 0.1003\,mol\,kg^{-1}$$

Acetic acid dissociates in water in accordance with the reaction,

	$CH_3COOH \rightleftharpoons$	CH_3COO^-	$+H^+$
Initial amount:	1	0	0
Amount at equilibrium:	$1-\alpha$	α	α

Total number of moles at equilibrium = $1 - \alpha + \alpha + \alpha = 1 + \alpha$

So, van't Hoff's factor, $i = \frac{1+\alpha}{1}$

We have, α = 23% = 0.23

Then, $i = \frac{1+0.23}{1} = 1.23$

$\Delta T_f = i \times K_f \times m$

$= 1.23 \times 1.86\,K\,kg\,mol^{-1} \times 0.1003\,mol\,kg^{-1}$

= 0.229 K

47. (A) Reactivity decreases as the magnitude of +ve charge on the carbonyl carbon decreases or the steric hindrance in the intermediate increases, i.e.,

$H_2C = O > RCHO > ArCHO > R_2C = O > Ar_2C = O$

48. (C) Rate $\propto$ [A] $[B]^0$ $[C]^2$. Hence, order = 3.

49. (D) NO_2 group withdraws electrons from o- and p-positions and hence activates the Cl towards nucleophilic substitution reactions.

50. (A) The net flow of the solvent is from dilute solution to concentrated solution.

51. (A) $Cr^{3+} + e^- \longrightarrow Cr^{2+}$, E° = – 0.41 volts

and

$Mn^{3+} + e^- \longrightarrow Mn^{2+}$, E° = + 1.51 volts

E° values show that Cr^{2+} is unstable and has a tendency to acquire more stable Cr^{3+} state by acting as a reducing agent. On the other hand Mn^{3+} is unstable and is reduced to more stable Mn^{2+} form.

52. (A) Only 1° alkyl halides, i.e., CH_3Br undergoes S_N2 reaction.

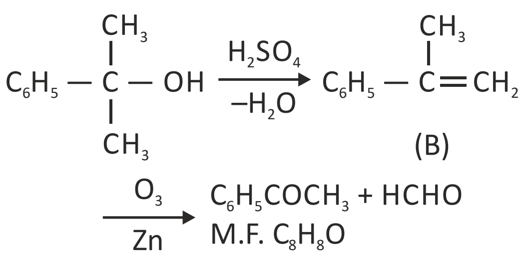
53. (A) As the standard reduction potentials of MnO_4^- (aq) + $8H^+$ (aq) + $5e^- \rightarrow Mn^{2+}$ + $4H_2O$ (l) and Cl_2 (g) + $2e^- \rightarrow 2Cl^-$ (aq) are almost of the same order, MnO_4^- cannot be used for quantitative estimation of aqueous $Fe(NO_3)_2$.

54. (B) $K = k_1 \times k_2 = (6.8 \times 10^{-3}) \times (1.6 \times 10^{-3}) = 1.08 \times 10^{-5}$

55. (A) As ketone with M.F. C_8H_8O shows +ve iodoform test, therefore, it must be a methyl ketone, i.e., $C_6H_5COCH_3$. As this ketone is obtained by the ozonolysis of an olefin (B) which is obtained by the addition of excess of CH_3 MgBr to an ester (A) with M.F. $C_9H_{10}O_2$, therefore, ester

(A) is $C_6H_5COOC_2H_5$ and the olefin (B) is $C_6H_5C(CH_3) = CH_2$ as explained below :

$$\underset{A(\text{M.F. } C_9H_{10}O_2)}{C_6H_5COOC_2H_5} \xrightarrow[\text{(ii) } H^+/H_2O]{\text{(i) } 2CH_3MgBr}$$



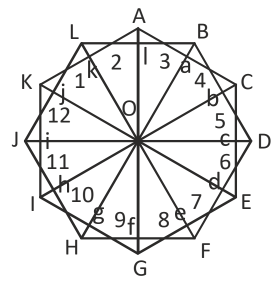
CRITICAL THINKING

56. (A) JOL, LOB, BOD, DOF, FOH, HOJ = 3×6=18

KOA, AOC, COE, EOG, GOI, IOK = 3×6=18

12K1, 1L2, 2A3, 3B4, 4C5, 5D6, 6E7, 7F8, 8G9, 9H10, 10I11, 11J12 = 12 × 3 = 36

18 + 18 + 36 = 72



Aoa, Bob, Coc, Dod, Eoe, Fof, Gog, Hoh, Ioi, Joj, Kok, Lol

72 + 12 = 84

57. (C) According to the passage, 'Last winter 50% of all fatal road accidents involved drivers with upto 5 years driving experience and an additional 15% were drivers who had between 6 to 8 years of experience.

This piece of data only mentions experience, not age. Although the main idea of the passage is that younger drivers are generally more likely to be involved in fatal car accidents, we cannot assume all relatively inexperienced drivers are young.

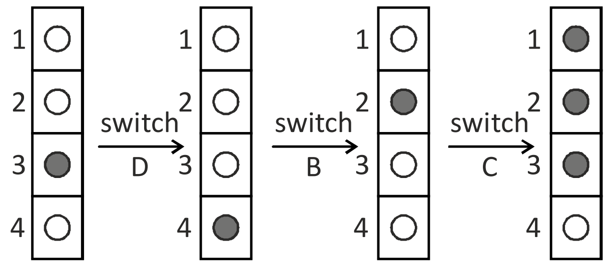
We do not know how many of those 15% with 6 to 8 years of experience are younger drivers and how many are older drivers.

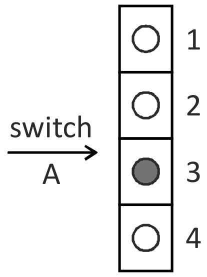
As this comparison is impossible to make on the basis of the information provided in the passage, the answer is cannot say.

58. (D) The given two statements are effects of two independent causes.

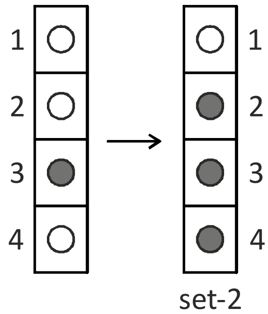
59. (A) The end supporting the punctured balloon tips upward as it is lightened by the weight of air that escapes. Although there is a loss of byouant force of the punctured balloon, that decrease in upward force is less than the weight of air loss, since the density of air in the balloon before puncturing was greater than the density of surrounding air.

60. (B) Switch B is faulty





D, B, C and A thrown in turn set - 1 chaned to set - 2. But the result figure



lights 2 and 4 are fault.

Hence, switch (B) is fault.

The End